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AQA

General Certificate of Education (A-level)
June 2012

Mathematics MPC1

(Specification 6360)

Pure Core 1

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

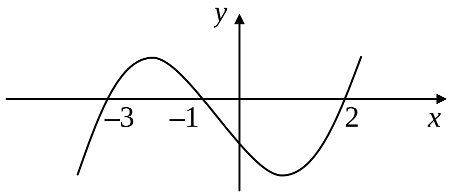
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1	$\dfrac{5\sqrt{3}-6}{2\sqrt{3}+3} \times \dfrac{2\sqrt{3}-3}{2\sqrt{3}-3}$	M1		
	(Numerator =) $30-15\sqrt{3}-12\sqrt{3}+18$	m1		correct $\left(=48-27\sqrt{3}\right)$
	(Denominator $=12-9=$) 3	B1		must be seen as denominator
	$\left(\dfrac{48-27\sqrt{3}}{3}\right)=16-9\sqrt{3}$	A1	4	CSO; accept $16+-9\sqrt{3}$
	Total		**4**	
2(a)(i)	$y=\dfrac{4}{3}x-\dfrac{7}{3}$	M1		$y=\pm\dfrac{4}{3}x+k$ or $\dfrac{\Delta y}{\Delta x}$ with 2 **correct** points
	$\Rightarrow$ grad $AB=\dfrac{4}{3}$	A1	2	condone slip in rearranging if gradient is correct; condone 1.33 or better
(ii)	$y=$ 'their grad' $x+c$ **and** attempt to use $x=3,\ y=-5$	M1		***or*** $y--5=$ 'their grad AB' $(x-3)$ ***or*** $4x-3y=k$ and attempt to find k using $x=3$ and $y=-5$
	$y+5=\dfrac{4}{3}(x-3)$ *or* $y=\dfrac{4}{3}x-\dfrac{27}{3}$	A1		correct equation in any form but must simplify $--$ to +
	$4x-3y=27$	A1	3	integer coefficients in required form eg $-8x+6y=-54$
(b)	$4x-3y=7$ and $3x-2y=4$ $\Rightarrow 8x-9x=14-12$ etc	M1		must use **correct pair** of equations and **attempt** to eliminate x or y (generous)
	$x=-2$	A1		
	$y=-5$	A1	3	or $D\ (-2,-5)$
(c)	$4(k-2)-3(2k-3)=7$ $4k-8-6k+9=7$	M1		sub $x=k-2,\ y=2k-3$ into $4x-3y=7$ and attempt to multiply out with all k terms on one side (condone one slip)
	$\Rightarrow k=-3$	A1	2	
	Total		**10**	

MPC1

Q	Solution	Marks	Total	Comments
3(a)(i)	$\text{p}(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$	M1		$\text{p}(-1)$ attempted **not** long division
	$\text{p}(-1) = -1 + 2 + 5 - 6 = 0 \Rightarrow x + 1$ is a factor	A1	2	CSO; correctly shown = 0 plus statement
(ii)	Quad factor in this form: $(x^2 + bx + c)$	M1		long division as far as constant term ***or*** comparing coefficients, ***or*** $b = 1$ ***or*** $c = -6$ by inspection
	$x^2 + x - 6$	A1		correct quadratic factor
	$[\text{p}(x) =]\ (x+1)(x+3)(x-2)$	A1	3	**must** see correct product
(b)	$\text{p}(0) = -6$; $\text{p}(1) = -8$	M1		**both** $\text{p}(0)$ and $\text{p}(1)$ attempted **and** at least one value correct
	$\Rightarrow \text{p}(0) > \text{p}(1)$	A1	2	**AG** both values correct plus correct statement involving p(0) and p(1)
(c)		M1 A1 A1	3	cubic with one max and one min ∩∪ with –3, –1, 2 marked correct with minimum to right of *y*-axis AND going beyond –3 and 2
	Total		**10**	

MPC1

Q	Solution	Marks	Total	Comments
4(a)(i)	$3x^2+3x^2+xy+xy+3xy+3xy$	M1		correct expression for surface area
	$6x^2+8xy=32$			$2(3x^2+xy+3xy)=32$ etc
	$\Rightarrow 3x^2+4xy=16$	A1	2	**AG** be convinced
(ii)	$(V=)3x^2y$ OE	M1		correct volume in terms of x and y
	$=3x\left(\frac{16-3x^2}{4}\right)$ **or** $=3x^2\left(\frac{16-3x^2}{4x}\right)$			OE
	$=12x-\frac{9x^3}{4}$	A1	2	CSO **AG** be convinced that all working is correct
(b)	$\left(\frac{dV}{dx}=\right)12-\frac{27}{4}x^2$	M1 A1	2	one of these terms correct all correct with 9 × 3 evaluated (no + c etc)
(c)(i)	$x=\frac{4}{3}\Rightarrow\frac{dV}{dx}=12-\frac{27}{4}\times\left(\frac{4}{3}\right)^2$	M1		attempt to sub $x=\frac{4}{3}$ into '*their*' $\frac{dV}{dx}$
	$\frac{dV}{dx}=12-\frac{27}{4}\times\frac{16}{9}=12-12$			or $12-\frac{432}{36}=12-12$ or $12-\frac{48}{4}=0$ etc
	$\frac{dV}{dx}=0\Rightarrow$ stationary value	A1	2	CSO; shown = 0 plus statement
(ii)	$\frac{d^2V}{dx^2}=-\frac{27x}{2}$ OE	B1√		FT for '*their*' $\frac{dV}{dx}=a+bx^2$
	when $x=\frac{4}{3}$, $\frac{d^2V}{dx^2}<0\Rightarrow$ maximum	E1√	2	**or** sub of $x=\frac{4}{3}$ into '*their*' $\frac{d^2V}{dx^2}$ $\Rightarrow$ maximum
	$\left(\text{FT "}\textit{minimum}\text{" if their } \frac{d^2V}{dx^2}>0\right)$			E0 if numerical error seen
	Total		**10**	

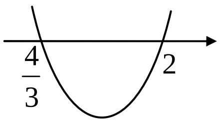
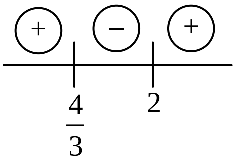
MPC1

Q	Solution	Marks	Total	Comments
5(a)(i)	$\left(x-\frac{3}{2}\right)^2$	M1		or $p = 1.5$ stated
	$\left(x-\frac{3}{2}\right)^2 + \frac{11}{4}$	A1	2	$(x-1.5)^2 + 2.75$
	Mark their final line as their answer			
(ii)	$x = \frac{3}{2}$	B1√	1	correct or FT their " $x = p$ "
(b)(i)	$x^2 - 3x + 5 = x + 5 \Rightarrow x^2 = 4x$	M1		eliminating x or y and collecting like terms (condone **one** slip) **or** $(y-5)^2 - 3(y-5) + 5 = y$ $\Rightarrow y^2 - 14y + 45 = 0$
	$(x \neq 0)$ $\Rightarrow x = 4$	A1		
	$y = 9$	A1	3	
(ii)	$\frac{x^3}{3} - \frac{3x^2}{2} + 5x\ (+c)$	M1 A1 A1	3	one of these terms correct another term correct all correct (need not have $+c$)
(iii)	$[\ ]_0^4 = \frac{4^3}{3} - 3\times\frac{4^2}{2} + 5\times 4$	M1		must have earned M1 in part(b)(ii) $\mathrm{F}(\text{their } x_B)\ \{-\mathrm{F}(0)\}$ "correctly sub'd"
	$= 17\frac{1}{3}$	A1		$\left(\frac{64}{3} - 24 + 20 =\right)$ $\frac{52}{3}$ or $\frac{104}{6}$ etc condone 17.3 but not $16\frac{4}{3}$ etc
	Area trapezium $= \frac{1}{2}(x_B)(5 + y_B)$	B1√		FT their numerical values of x_B, y_B Area = $\frac{1}{2}\times 4\times 14$ (= 28)
	Area of shaded region $= 28 - 17\frac{1}{3}$			
	$= 10\frac{2}{3}$	A1	4	CSO; $\frac{32}{3}$, accept 10.7 or better
	Total		**13**	

MPC1

Q	Solution	Marks	Total	Comments
6(a)	$(x-5)^2+(y-8)^2$	B1		
	$=25$	B1	2	condone 5^2
(b)(i)	$(2-5)^2+(12-8)^2$			or $AC^2=3^2+4^2$
	$=9+16 \quad =25$			hence $AC=5$; (also radius = 5)
	$\Rightarrow$ A lies on circle	B1	1	CSO
				$(\Rightarrow$ radius $=AC)$ $\Rightarrow$ A lies on circle
	(must have concluding statement and circle equation correct if using equation)			(must have concluding statement & RHS of circle equation correct or r =5 stated if Pythagoras is used)
(ii)	grad $AC=-\frac{4}{3}$	B1		
	Gradient of tangent is $\frac{3}{4}$	B1√		FT their –1/ grad AC
	$y-12=$ '*their tangent grad*' $(x-2)$	M1		**or** $y=$ '*their tangent grad*' $x+c$ & attempt to find c using $x=2$, $y=12$
	$y-12=\frac{3}{4}(x-2)$ ***or*** $y=\frac{3}{4}x+\frac{21}{2}$ etc	A1		correct equation in any form
	$3x-4y+42=0$	A1	5	CSO; must have integer coefficients with all terms on one side of equation accept $0=8y-6x-84$ etc
(c)(i)	$(CM^2=)$ $(7-5)^2+(12-8)^2$	M1		**or** $(CM^2=)$ 20
	$(\Rightarrow CM=\sqrt{20})$ $\Rightarrow (CM=)$ $2\sqrt{5}$	A1	2	
(ii)	$PM^2=PC^2-CM^2 \quad =25-20$	M1		Pythagoras used correctly
				eg $d^2+\left(2\sqrt{5}\right)^2=5^2$
	$\Rightarrow PM=\sqrt{5}$	A1		
	Area $\Delta\ PCQ=\sqrt{5}\times 2\sqrt{5}$			
	$=10$	A1	3	CSO
	Total		**13**	

MPC1

Q	Solution	Marks	Total	Comments
7(a)(i)	$(\text{Increasing} \Rightarrow) \dfrac{dy}{dx} > 0$ $20x - 6x^2 - 16 > 0$ either	M1		correct interpretation of y increasing
	$\Rightarrow 6x^2 - 20x + 16 < 0$ *or* $(2)\ (10x - 3x^2 - 8) > 0$			must see at least one of these steps before final answer for A1
	$\Rightarrow 3x^2 - 10x + 8 < 0$	A1	2	CSO **AG** no errors in working
(ii)	$(3x-4)(x-2)$	M1		correct factors or correct use of quadratic equation formula as far as $\dfrac{10 \pm \sqrt{4}}{6}$
	CVs are $\dfrac{4}{3}$ and 2	A1		condone $\dfrac{8}{6}$ and $\dfrac{12}{6}$ here but not in final line
	 	M1		sketch or sign diagram
	$\dfrac{4}{3} < x < 2$	A1	4	or $2 > x > \dfrac{4}{3}$ accept $x < 2$ **AND** $x > \dfrac{4}{3}$ but **not** $x < 2$ **OR** $x > \dfrac{4}{3}$ **nor** $x < 2\ ,\ x > \dfrac{4}{3}$
	Mark their final line as their answer			

MPC1

Q	Solution	Marks	Total	Comments
7(b)(i)	$x = 2$; $\left(\frac{dy}{dx} =\right)$ $40 - 24 - 16$	M1		sub $x = 2$ into $\frac{dy}{dx}$ and simplify terms
	$\frac{dy}{dx} = 0 \Rightarrow$ tangent at P is parallel to the x-axis	A1	2	must be all correct working plus statement
(ii)	$x = 3$; $\frac{dy}{dx} = 20\times3 - 6\times3^2 - 16$	M1		must attempt to sub $x = 3$ into $\frac{dy}{dx}$
	(= 60–54–16) $= -10$	A1		
	Gradient of normal $= \frac{1}{10}$	A1√		$\frac{-1}{\text{"their } -10\text{"}}$
	Normal: $(y - -1) =$ 'their grad'$(x - 3)$	m1		normal attempted with correct coordinates used and gradient obtained from their $\frac{dy}{dx}$ value
	$y + 1 = \frac{1}{10}(x - 3)$	A1		any correct form, eg $10y = x - 13$ but must simplify – – to +
	(Equation of tangent at P is) $y = 3$	B1		
	$x = 43$	A1	7	CSO; $\Rightarrow R\,(43, 3)$
	Total		**15**	
	TOTAL		**75**	